PREDICTING STATIC BENDING MODULUS OF ELASTICITY OF TROPICAL AFRICAN HARDWOODS FROM DENSITY USING A MODEL BASED ON LONGITUDINAL VIBRATION

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ABSTRACT
The longitudinal vibration technique was examined as a means of predicting static bending modulus of elasticity (MOE) from wood density of tropical African hardwoods. Dynamic MOEs measured using the longitudinal vibration test of large specimens of Obeche (Triplochiton scleroxylon), Makore (Tieghemella hekellii) and Mobi (Baillonella toxisperma) were 19.6 and 12% respectively higher than static bending MOEs reported in the literature. Dynamic MOE was strongly correlated to wood density (r = 0.97), and a linear regression model developed could predict static bending MOE from wood density when tested on some 42 commercial and secondary tropical African hardwoods, with percentage errors ranging up to 17%. In view of the lack of proper laboratory wood testing machines in tropical developing African countries, the model is recommended as a useful and fast tool for predicting static modulus of elasticity of tropical timbers, especially the secondary species, from their wood densities. It may also be applicable in the finger-jointing industry for sorting and matching random short lengths of timber for jointing together. If properly applied, the model is expected to lend support to sustainable tropical forest management and the efficient utilization of tropical timber resources.

Keywords: Bending modulus of elasticity, Tropical hardwoods, Longitudinal vibration

INTRODUCTION
The tropical rainforests in many African countries are disappearing under onslaughts of illegal logging, bush fires and pollution. Population pressures have also led to a marked decline in closed forest areas because of shifting cultivation on a decreasingly shorter cycle and sometimes because of requirements for fuelwood (Oldeman, 1982; Ghana Forestry Department, 1994; Ministry of Lands and Forestry, Ghana, 1996). In West Africa, the closed tropical high forests have been predicted to shrink from 14 to 7.5 million ha by the year 2000 (Oldeman, 1982).

Ghana’s closed high forest zone, covering about 8.2 million ha, at the turn of the century, has diminished to the present level of about 1.6 million ha, partly due to inefficient logging and wood processing methods (Food and Agriculture Organization of the United Nations, 1997; Ministry of Lands and Forestry, Ghana, 1996). Out of about 420 timber species growing to exploitable sizes in the forest of Ghana, only about 64 species are presently being exploited as commercial timber species. The rest, the secondary species, are unexploited due to lack of reliable technical information on their properties and possible areas of utilization (Ghartey, 1989; Ministry of Lands and Forestry, Ghana, 1996). As a result of the selective timber exploitation, 32 of the major economic species are already under imminent threat of economic extinction (Ministry of Lands and Forestry, Ghana, 1996). There is, therefore, an urgent need to determine the mechanical properties of the secondary species to justify their possible substitution for the diminishing economic species.

In Ghana’s relatively large timber processing industry, large volumes of wood residue or waste are generated, most of which are reportedly suitable for the production of high value-added products such as finger-jointed timber (Prah, 1994; Ofosu-Asiedu, Nani-Nutakor & Ayarkwa, 1996). It is presently estimated that timber recovery in Ghanaian sawmills ranges between 40 and 50%, and the remaining generated as sawmill residues. The utilization of solid sawmill residues is expected to bring economic benefits to the country, enhance the productivity of timber processing mills and also lend support to sustainable forest management. For finger-jointing, however, the short random lengths are expected to be sorted out and matched on the basis of modulus of elasticity (MOE), to narrow the variability in strength properties of the random pieces and produce products of uniform stiffness profile, with great potential for machine stress rated (MSR) lumber grades. Determination of reliable mechanical properties of wood, or the mechanical stress grading of wood requires expensive test equipment, which is difficult to come by in many developing tropical African countries (Addae-Mensah, Ayarkwa, Mohammed & Azerongo, 1989). Mechanical wood test-
The general equation relating wavelength ($\lambda$) to length of the rod (L) and mode of vibration (n) is given by

$$\lambda = \frac{2L}{n} \quad \text{2}$$

The wavelength is the ratio of wave velocity (C) to the frequency of vibration (f).

Thus,

$$\lambda = \frac{C}{f} \quad \text{3}$$

Combining the three equations gives an expression for the frequency of vibration as follows:

$$f = \frac{n\sqrt{E/\rho}}{2L} \quad \text{4}$$

Rearranging equation 4 and introducing weight per unit volume and gravitational acceleration, in order to obtain MOE with units of force per unit area, gives for the fundamental frequency ($n=1$),

$$E = \left(\frac{4W L f}{A g}\right) 9.8 \times 10^{-5} \quad \text{5}$$

where

- $E$ = modulus of elasticity, MOE (GPa)
- $W$ = weight of timber (kgf)
- $L$ = length of timber (cm)
- $f$ = fundamental resonant frequency (s$^{-1}$)
- $A$ = cross-sectional area of timber sample (cm$^2$)
- $g$ = gravitational acceleration (cm/s$^2$)

Thus, the dynamic modulus of elasticity of a timber piece can be determined from its length, weight, cross-sectional area, fundamental resonant frequency and the gravitational acceleration.

**MATERIALS AND METHODS**

**Preparation of Specimens**

Wood samples of the following three tropical hardwoods of low, medium and high densities respectively were used for the study:

- Obeche (Triplochiton scleroxylon)
- Makore (Tieghemella heckelii Pierre)
- Moabi (Bailonnella toxasperma Pierre)

Radial-sawn, straight-grained heartwood samples (Fig. 1) were randomly collected from three logs of each species. The samples were planed and cross cut to final dimensions (Table 1) such that no visible defects such as knots and spiral grains were present, and growth layers were at right angle to the width of each specimen. The kiln-dried samples were conditioned to about 8% moisture con-
Predicting MOE of tropical African hardwoods from density

J. Ayarkwa et al.

tent for the test. In all 638 specimens were tested.

TABLE 1
Quantities and dimensions of timber test specimens

<table>
<thead>
<tr>
<th>Timber species</th>
<th>Test specimens</th>
<th>Dimensions (mm)</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obeche (Triplochiton scleroxylon)</td>
<td>23x155x1000</td>
<td>210</td>
<td></td>
</tr>
<tr>
<td>Makore (Tieghemella heckellii)</td>
<td>23x155x1000</td>
<td>218</td>
<td></td>
</tr>
<tr>
<td>Moabi (Baiillonella toxisperma)</td>
<td>23x130x1000</td>
<td>210</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>638</td>
<td></td>
</tr>
</tbody>
</table>

Testing Methods

Wood Density
Wood density was determined in accordance with ASTM D2395-93. The mass of each specimen was determined using an electronic weighing balance, and the volume determined from the exact dimensions of each specimen measured at three locations along the specimen and averaged to ensure accuracy. The density was calculated as the ratio of mass to volume of each specimen.

Dynamic Modulus of Elasticity (MOE)
The dynamic modulus of elasticity was determined by the longitudinal vibration test. The technique involved introducing vibration into the test specimen by mechanical impact using a hammer, and the vibration received by a microphone, which transmitted the sound waves into a Frequency (FFT) Analyzer (Fig. 2). Peak frequency of 5kHz was selected. The fundamental resonance frequency was then measured by the FFT Analyzer, to an accuracy of 1Hz, and recorded. The modulus of elasticity of the specimen was then calculated using Formula 5. For comparison with data in the literature, the test data were corrected to 12% moisture content before the analyses, in accordance with ASTM D2915-94 as follows:

\[
P_2 = P_1 \frac{(\alpha - \beta M_1)}{\frac{1}{\alpha - \beta M_1}}
\]

where

- \(P_1\) = MOE measured at moisture content \(M\)
- \(P_2\) = MOE adjusted to moisture content \(M\), \(M_1, M_2\) = moisture contents (%)
- \(\alpha, \beta\) = moisture content constants, 1.44 and 0.02 respectively

Model Development
The mechanical properties of clear wood are generally linearly related to wood density (Kollman & Cote, 1968; Forest Products Laboratory, 1987; Bodig & Jayne, 1982; Bucur, 1995). Least squares regression analyses were therefore performed to correlate dynamic modulus of elasticity to the density of each wood species and for the combined data of the three species. The models used in the regression analyses were of the following form:

For Obeche, \(E_1 = \beta_0 + \beta_1 D_1 + \epsilon\)
For Makore, \(E_2 = \beta_0 + \beta_1 D_2 + \epsilon\)
For Moabi, \(E_3 = \beta_0 + \beta_1 D_3 + \epsilon\)

For the three species, \(E = \beta_0 + \beta_1 D + \epsilon\)

where \(E_1, E_2, E_3\) and \(E\) = dynamic MOE of Obeche, Makore, Moabi and any tropical African hardwood respectively
\(D_1, D_2, D_3\) and \(D\) = wood density of Obeche, Makore, Moabi and any tropical African hardwood respectively
\(\beta_0, \beta_1\) = regression coefficients
\(\epsilon_1, \epsilon_2, \epsilon_3\) and \(\epsilon\) = residual errors

RESULTS AND DISCUSSION
Table 2 presents a summary of test results showing the mean values, coefficients of variation and maximum and minimum values for dynamic modulus of elasticity (MOE) and wood density for the three species, with percentage errors from standard static test values taken from literature (Takahashi, 1978).

Mean values of dynamic MOE obtained from the
test were slightly higher than that from standard static test values taken from the literature. Percentage errors of about 12, 6 and 19% were obtained for Moabi, Makore and Obeche respectively. The accuracy achieved compares well with what is reported in the literature (Kollman & Cote, 1968; Bodig & Jayne, 1982; Tsoumis, 1991; Bucur, 1995).

Equations of the Predicting Models
Plots of the regression of dynamic MOE on density for Obeche, Makore and Moabi are shown in Fig. 3, 4 and 5 respectively, and that for the combined data for the three species is shown in Fig. 6. Table 3 shows equations of the models as well as correlation coefficients between dynamic MOE and density for each species and also for the combined data. Makore test samples of very low densities and corresponding MOEs were not included in the regression analyses.

The correlation coefficients between dynamic MOE and density for Obeche, Makore and Moabi were 0.39, 0.69 and 0.68 respectively (Table 3). When data from the three species were combined, a correlation coefficient of 0.97 was obtained. The results, thus indicate positive linear correlation between density and dynamic MOE and agree well with the literature (Kollman & Cote, 1968; FPL, 1981; Bodig & Jayne, 1982; Bucur, 1995). Kollman & Cote (1968) reported that linear relationships were obtained between modulus of elasticity and density, from vibration tests for Spruce and Oak. The coefficient of determination calculated for the combined experimental data \( R^2 = 0.94 \) indicates that only about 8% of the variability of predicted MOE is not accounted for by the wood density, indicating the adequacy of the model. The seemingly low correlation obtained for the relatively low density Obeche may be due to the difficulty of identifying any severe cross-grained samples in the pale white coloured wood. This may explain the two-tier pattern of the scatter plot of the data points (Fig. 3). The results seem to indicate a good correlation between dynamic MOE and the medium density Makore and high density Moabi. This may be explained by the comparatively very straight-grained nature of the wood of both species. Dynamic modulus of elasticity strongly correlated with density when the three species were combined, possibly due to the large sample size. The regression model obtained for the combined data represents species of low, medium and high densities, comprising the majority of tropical African hardwoods. The discontinuity in the pooled data (Fig. 6), that is, between Obeche and Makore data points, could have been removed and the regression models enhanced if another wood species of density ranging between 460 and 650 kg/m³ had been tested. However, this was not possible under the experiment.
TABLE 3
Results of the regression of dynamic MOE on density of three tropical hardwoods

<table>
<thead>
<tr>
<th>Timber species</th>
<th>Sample size</th>
<th>Model equation</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oboche</td>
<td>210</td>
<td>( E_1 = 3.8795 + 0.0093D )</td>
<td>0.3865</td>
</tr>
<tr>
<td>Makore</td>
<td>218</td>
<td>( E_2 = -0.7215 + 0.0226D )</td>
<td>0.6917</td>
</tr>
<tr>
<td>Moabi</td>
<td>210</td>
<td>( E_3 = -7.5371 + 0.0303D )</td>
<td>0.6764</td>
</tr>
<tr>
<td>All species</td>
<td>638</td>
<td>( E = -0.5841 + 0.0215D )</td>
<td>0.9674</td>
</tr>
</tbody>
</table>

is not linearly related to wood density is rejected at 5% significance level.

Test of Fitness of the Combined Linear Model

In assessing the fit of the linear model, the residuals which measure the unknown model errors were used. It was verified whether the initial model assumptions are fulfilled; that is, residuals are independently, identically and normally distributed, and are independent of the explanatory and response variables (i.e., wood density and dynamic MOE respectively). According to Kottegoda & Rosso (1998), graphical methods usually provide confirmation that there are no shortcomings or systematic defects in the model.

The normal probability plot (Fig. 8) does not indicate any departures from normality in the residuals, as there are no heavy-tailed distribution, outliers or any ontoward behaviour. The break in the graph is the result of the discontinuity of the range of densities studied. The graph shows that the distribution is close to normality.

The index plot of the residuals against observation numbers (Fig. 7) does not show any significant autocorrelation in the residuals.

The dispersion of the data points in the plot of residuals against wood densities as well as against the predicted MOEs (Fig. 9 and 10) indicates that the errors, as represented by the residuals, are independent of the explanatory and response variables. Thus, the linearity assumption holds. It is also reasonable to assume that the variance of the

Analysis of Variance (ANOVA) of the Regression

The results of the ANOVA of the regression (Table 4) show that F value is 9049.97, which is far greater than the critical value \( F_{1, 636} 0.05 = 3.85 \). Therefore the null hypothesis that dynamic MOE
Predicting MOE of tropical African hardwoods from density distribution of the residuals is constant, as there are not much larger spread above and below one part of the horizontal- or zero-axis than another (Fig. 9 and 10).

Combined data from the three species was tested using the entire list of 40 important commercial hardwoods contained in Bulletin No. 9 of the Forest Products Research Institute of Ghana (Addae-Mensah et al., 1989) as well as two other species. Dynamic MOE of each of the species was estimated from the model by substituting the species density (specified in the bulletin), and the results compared with the standard static MOE in the literature based on three point loading (Takahashi, 1978; Addae-Mensah et al., 1989). Absolute percentage error of each predicted value was calculated as follows:

\[
\text{Absolute percentage error} = \frac{\text{estimated MOE} \ - \ \text{static bending MOE}}{\text{static bending MOE}} \times 100
\]

The results (Table 5) indicate that estimated dynamic MOE differ only slightly from the standard static MOE reported in the literature (Takahashi, 1978; Addae-Mensah et al., 1989). The predicted values were generally higher, and in some few cases lower, than the literature values. The absolute percentage error, however, ranged up to about 17%. This range of differences seems reasonable in view of the quite variable mechanical properties of wood even from the same log. The model could thus predict static bending MOE within the same level of accuracy as obtained from the actual experimental test of dynamic MOE. Previous experimental data available also indicate that dynamic MOE is generally higher than standard static MOE by up to about 19% for Spruce, and this was considered small and negligible (Kollman & Cote, 1968). Thus, the accuracy of the prediction using the developed model seems high and acceptable. The static modulus of elasticity of any tropical hardwood ranging from low, medium to high density may therefore be predicted using the model to an accuracy varying up to about 17%.
Predicting MOE of tropical African hardwoods from density

J. Ayarkwa et al.

**CONCLUSION AND RECOMMENDATIONS**
Dynamic MOE was strongly correlated to wood density \( (r=0.97) \) for the combined data from the three tropical African hardwoods. Static bending MOEs of 42 commercial and secondary tropical African species could be predicted from their wood densities, using the model \( E = 0.0215D - 0.5481 \), with absolute percentage errors of up to 17%, which is within acceptable limits.

Although bending test is generally recognized as...
a more desirable method of determining MOE, the developed model could be useful for predicting static MOE from density in situations where it is not feasible to conduct bending test. If the model is used, however, wood defects such as spiral grains and knots should be excluded from samples for density measurements.

The model is recommended as a useful and fast tool for predicting bending MOE, and hence the mechanical wood quality of tropical African hardwoods, especially the secondary species. It may also be applicable in the finger-jointing industry for sorting and matching random short timber pieces, on the basis of MOE, for jointing together. What is required is only a simple wood density measurement.

If properly utilized, the model may lend support to future mechanical stress grading and efficient utilization of tropical African hardwoods, and also to sustainable tropical rain forest management.

REFERENCES